

Exercise set 2.2

Solving systems of equations using matrices

Here are some problems for you to try your hand at.

1. Use matrices to solve the system of equations:

$$\begin{aligned} x - 3y &= 7 \\ -3x + 4y &= -1 \end{aligned}$$

1st pivot $\rightarrow$

$$\begin{bmatrix} 1 & -3 & 7 \\ -3 & 4 & -1 \end{bmatrix} \begin{array}{l} \text{Already 1!} \\ R_2 + 3R_1 \end{array}$$

$$\begin{bmatrix} 1 & -3 & 7 \\ 0 & -5 & 20 \end{bmatrix} \begin{array}{l} R_1 + 3R_2' \\ R_2 \div -5 \end{array}$$

2nd pivot $\rightarrow$

$$R_2' \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \end{bmatrix}$$

$$\therefore \begin{aligned} x &= -5 \\ y &= -4 \end{aligned}$$

2. Use matrices to solve the system of equations:

$$\textcircled{1} \quad 2x + 1 - 3y = -2y - 1$$

$$\textcircled{2} \quad 3x - y + 4 = 5 + 2x$$

First, Organise the equations:

$$\begin{aligned} \textcircled{1} \quad 2x - 3y + 2y &= -1 - 1 \\ 2x - y &= -2 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 3x - 2x - y &= 5 - 4 \\ x - y &= 1 \end{aligned}$$

Now form the matrix:

$$\begin{bmatrix} 2 & -1 & -2 \\ 1 & -1 & 1 \end{bmatrix} \begin{array}{l} R_1 \div 2 \\ R_2 - R_1' \end{array}$$

$$R_1' \begin{bmatrix} 1 & -1/2 & -1 \\ 0 & -1/2 & 2 \end{bmatrix} \begin{array}{l} R_1 + 1/2 R_2' \\ R_2 \times -2 \end{array}$$

$$R_2' \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix}$$

$$\therefore \begin{aligned} x &= -3 \\ y &= -4 \end{aligned}$$

3. Use matrices to solve the system of equations:

$$\begin{aligned} 3x - 6y + 9z &= 0 \\ 4x - 6y + 8z &= -4 \\ -2x - y + z &= 7 \end{aligned}$$

$$\begin{array}{cccc|l} x & y & z & & \\ \textcircled{3} & -6 & 9 & 0 & R_1 \div 3 \\ 4 & -6 & 8 & -4 & R_2 - 4R_1' \\ -2 & -1 & 1 & 7 & R_3 + 2R_1' \end{array}$$

$$R_1' \begin{array}{cccc|l} 1 & -2 & 3 & 0 & R_1 + 2R_2' \\ 0 & \textcircled{2} & -4 & -4 & R_2 \div 2 \\ 0 & -5 & 7 & 7 & R_3 + 5R_2' \end{array}$$

$$R_2' \begin{array}{cccc|l} 1 & 0 & -1 & -4 & R_1 + 4R_3' \\ 0 & 1 & -2 & -2 & - \\ 0 & 0 & \textcircled{-3} & -3 & R_3 \div 3 \end{array}$$

$$R_3' \begin{array}{cccc|l} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}$$

$$\therefore \begin{aligned} x &= -3 \\ y &= 0 \\ z &= 1 \end{aligned}$$

4. Use matrices to solve the system of equations:

$$\begin{aligned} 3x + 2y + 4z &= 600 \\ 2x + y + z &= 275 \\ 6x + 6y + 2z &= 1150 \end{aligned}$$

$$\begin{array}{cccc|l} x & y & z & & \\ \textcircled{3} & 2 & 4 & 600 & R_1 \div 3 \\ 2 & 1 & 1 & 275 & R_2 - 2R_1' \\ 6 & 6 & 2 & 1150 & R_3 - 6R_1' \end{array}$$

$$R_1' \begin{array}{cccc|l} 1 & 2/3 & 4/3 & 200 & R_1 - 2/3 R_2' \\ 0 & \textcircled{-1/3} & -5/3 & -125 & R_2 \times -3 \\ 0 & 2 & -6 & -50 & R_3 - 2R_2' \end{array}$$

$$R_2' \begin{array}{cccc|l} 1 & 0 & -2 & -50 & R_1 + 2R_3' \\ 0 & 1 & 5 & 375 & R_2 - 5R_3' \\ 0 & 0 & \textcircled{-16} & -800 & R_3 \div -16 \end{array}$$

$$\begin{array}{cccc|l} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 125 \\ 0 & 0 & 1 & 50 \end{array}$$

$$\therefore \begin{aligned} x &= 50 \\ y &= 125 \\ z &= 50 \end{aligned}$$

5. Use matrices to solve the system of equations:

$$3x - 3y = -12 - 15z$$

$$8z = -5 + y - 2x$$

$$-x + 3y - 8 = +8z$$

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$$3x - 3y + 15z = -12$$

$$2x - y + 8z = -5$$

$$-x + 3y - 8z = +8$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline \textcircled{3} & -3 & 15 & -12 \\ 2 & -1 & 8 & -5 \\ -1 & 3 & -8 & 8 \end{array} \begin{array}{l} R_1 \div 3 \\ R_2 - 2R_1' \\ R_3 + R_1' \end{array}$$

$$R_1' \begin{array}{ccc|c} 1 & -1 & 5 & -4 \\ 0 & \textcircled{1} & -2 & 3 \\ 0 & 2 & -3 & 4 \end{array} \begin{array}{l} R_1 + R_2 \\ - \\ R_3 - 2R_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & \textcircled{1} & -2 \end{array} \begin{array}{l} R_1 - 3R_3 \\ R_2 + 2R_3 \\ - \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}$$

$$\therefore \begin{array}{l} x = 5 \\ y = -1 \\ z = -2 \end{array}$$