

1. Walk the line¹ (10 marks)

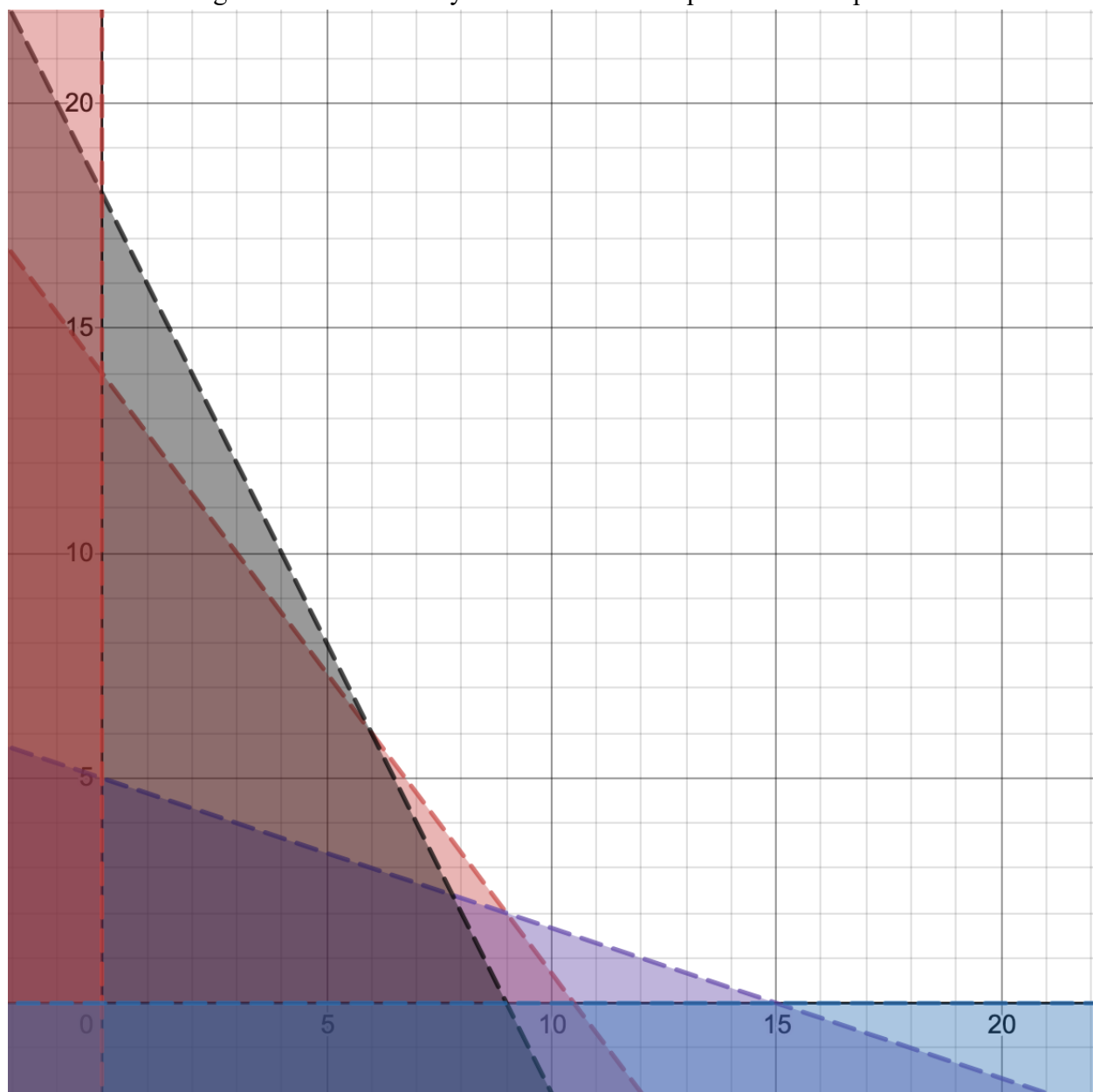
We saw that when we apply the simplex method to a problem in standard form it walks around the feasible set from vertex to vertex until it reaches one representing the optimal solution. I wonder if it does the same for a problem in non-standard form? Remembering that “Every tableau represents a solution,” trace the path of the simplex solution to the NASA diet problem. Does it also walk the boundary of the feasible set, or does it do something else?

Here is our initial simplex tableau for the problem. (Sorry there are no interior dividers in the matrix, but MS Word refuses.)

$$\begin{array}{c}
 u \\
 v \\
 w \\
 W
 \end{array}
 \begin{array}{cccccc}
 A & B & u & v & w & W \\
 \left[\begin{array}{cccccc}
 -4 & -3 & 1 & 0 & 0 & 0 \\
 -2 & -6 & 0 & 1 & 0 & 0 \\
 -2 & -1 & 0 & 0 & 1 & 0 \\
 3 & 2 & 0 & 0 & 0 & -1
 \end{array} \right. & \begin{array}{c} -42 \\ -30 \\ -18 \\ 0 \end{array}
 \end{array}$$

¹ Walk-the-line. Verb. (idiomatic) To maintain an intermediate position between contrasting choices, opinions, etc.

And here's an image of the feasible set you can trace the simplex method's path on.



2. Loosey-Goosey² (10 marks)

The simplex method for a problem in standard form is quite prescriptive: the most negative value in the left part of the bottom row tells us the pivot column, and then the lowest positive constant/coefficient ratio tells us the pivot value in that column. By comparison the method for problems in nonstandard form is loosey-goosey:

Pick **any** of the negative values in the upper right part of the constant column.

Then choose **any** of the negative values in its row.

That seems like an awful lot of discretion for a mathematical algorithm. Start from the initial simplex tableau in problem 1 to answer the following questions:

- a) Do all choices³ lead to the same solution? Try making different choices and see if they do.
- b) Do all choices lead to the same number of pivots? As in a) try making different choices and see. If they don't, can you say what makes a good choice?

Hint: A **lot** of choices are possible. You will save yourself a lot of time using the pivoting app!

² Loosey-goosey. Adjective. Imprecise, disorganized, or excessively relaxed.

³ *Hint:* A **lot** of choices are possible. You will save yourself a lot of time using [the pivoting app!](#)

3. Also loose (10 marks)

Even the simplex method for a problem in standard form is not *completely* prescriptive. It tells us that the most negative value in the left part of the bottom row should be the pivot column. But what if there is a tie for most negative value? Consider this tableau with a three-way tie:

$$\begin{array}{c|cccccc} & x & y & z & u & v & C \\ \hline u & 1 & -1 & 2 & 1 & 0 & 0 & 10 \\ v & 2 & 1 & 3 & 0 & 1 & 0 & 12 \\ C & -2 & -2 & -2 & 0 & 0 & 1 & 0 \end{array}$$

- Do all pivot column choices lead to the same solution? Try making different choices and see if they do.
- Do all choices lead to the same number of pivots? As in a) try making different choices and see. If they don't, can you say what makes a good choice?

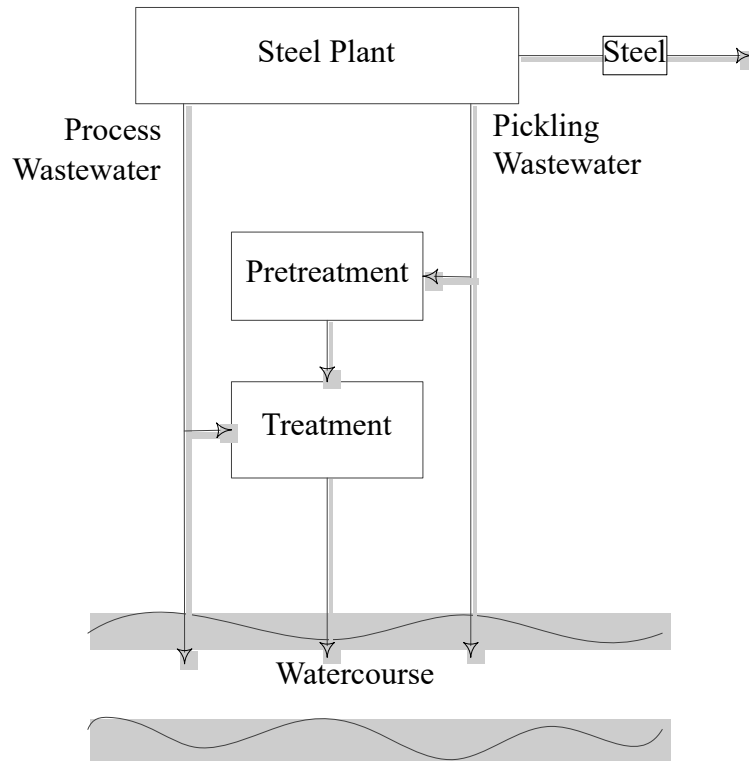
4. But what about forbs and grasses? (10 marks)

In formulating our LP model of Moose foraging behaviour we ignored their (possible) intake of Herbs (grass + forbs), because a reference informed us that “*Grasses rarely comprise more than 1% of the diet.*”⁴ But the moose we were modelling are not circumpolar moose (they were on Isle Royale in lake Superior), so perhaps their behaviour is different. Using a graphical analysis we couldn't model the ingestion of Herbs because it made our problem three dimensional, but with the simplex method this restriction is lifted. How much of the Isle Royale moose's diet does our model predict herbs will make up?

⁴ Schwartz, Charles C. *Physiological and nutritional adaptations of moose to northern environments*, ALCES Supplement 1 (192), pp. 139-155.

5. Corporate Behaviour (30 marks)

A rolling mill in a steel plant produces steel but also two kinds of wastewater: process water and pickling wastes. These wastes can either be discharged directly into a water course without treatment but subject to an effluent tax, or alternatively may be treated in an existing plant that removes 90 percent of the objectionable materials. The process wastewater can go directly into the treatment process, but pickling wastes require neutralization pretreatment before being treated. The system can be visualized like this:



Here are some data about the system (not current, this data is circa 1975):

- Income on steel manufactured: \$25/ton
- Pickle waste generated: 0.1 thousand gallons/ton of steel
- Process water generated: 1.0 thousand gallons/ton of steel
- Cost of pretreating pickle waste: \$0.20/thousand gallons
- Cost of treatment: \$0.10/thousand gallons
- Effluent tax on untreated pickle waste: \$0.15/thousand gallons
- Pretreatment plant capacity: 2,000,000 gallons/day
- Treatment plant capacity: 50,000,000 gallons/day
- Treatment plant efficiency: 0.90
- Regulations prohibit more than 500,000 gal/day of pickle waste or more than 10,000,000 gal/day of process water being discharged without treatment.

We formulated this as a linear programming problem to get the following:

$$\text{Maximize: } P = 24.985s - 0.15p - 0.1t$$

$$\text{Subject to: } p \leq 2,000$$

$$t + p \leq 50,000$$

$$s - t \leq 10,000$$

$$0.1s - p \leq 500$$

This problem is three dimensional (due to the three variables s , t and p) so we'll need to use the simplex method to solve it.

You have probably noticed that performing the simplex algorithm by hand is tiresome, but since it is precise and unambiguous the process can be automated and we can have a computer carry out either the pivoting for us, or find a complete sequence of tableaux leading to a solution.

The general pivot tool I have been using is

<https://www.zweigmedia.com/RealWorld/tutorialsf1/scriptpivot2.html> .

They also offer a simplex method tool at <https://www.zweigmedia.com/RealWorld/simplex.html> that will find all the simplex tableaux leading to a solution.

a) Whichever tool you use you should submit a complete sequence of properly labelled simplex tableaux that find the production plan to maximize profit.

b) Circle the pivot element in each tableau.

c) Interpret each tableau. "Interpret" in this context means to say how much steel is being produced, what the profit is, and what each of the slack variable values tell us about the system.

d) A journalist claims the plant is not treating as much water as they could. The company claims it is treating as much water as is possible. Who's right? (Assume the plant is operating to maximize profits in answering this.)

e) A local environmental group would like the steel plant to pollute less and is advocating doubling the effluent tax. Will this be effective in reducing the amount of pollutants reaching the waterway? Why, or why not? How would the doubling affect the company's profits?

f) Another group wants to forbid any untreated water from being released. Would this reduce corporate profits? If so, by how much?

g) Is it in the company's financial interest to increase the size either of its treatment plants? If so, how much should it be willing to spend to do so?