

Combining functions 2: Compositions

$$f(g(x))$$

$$\begin{matrix} \sin(x^2) \\ \uparrow \quad \uparrow \\ f' \quad g \end{matrix}$$

$$\boxed{\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)}$$

$$\text{eg. } \frac{d}{dx}[\sin(x^2)] = \cos(x^2) \cdot \frac{d}{dx}[x^2]$$

$$= \cos(x^2) \cdot 2x$$

derivative of
outer fn.

$\times$ derivative of
inner function.

$$\text{eg. } y = \underbrace{(x^3+x)}_g \overset{f}{^{\uparrow 10}}$$

$$f(x) = x^{10}$$

$$g(x) = x^3+x$$

$$f'(x) = 10x^9$$

$$g'(x) = 3x^2+1$$

$$y' = 10(x^3+x)^9 \cdot \frac{d}{dx}[x^3+x]$$

$$= 10(x^3+x)^9 \cdot (3x^2+1)$$

$$\text{eg. } y = \underbrace{e^{\sin x}}_f \overset{g}{\uparrow}$$

$$f(x) = e^x$$

$$g(x) = \sin x$$

$$f'(x) = e^x$$

$$g'(x) = \cos x$$

$$y' = f'(g(x)) \cdot g'(x)$$

$$= e^{\sin x} \cdot \cos x$$

$$\text{eg. } y = \sin(e^{x^5})$$

$$y' = \cos(e^{x^5}) \cdot e^{x^5} \cdot 5x^4$$