

1. (10 marks) **Emissions Reduction** The cost of controlling emissions often rises rapidly as the amount of emissions reduced increases, i.e. it is cheap and easy to find the first reductions, but becomes increasingly difficult and expensive to find further reductions. Here is a possible model for cost, $C(r) = 40 + 10r^2$ where r is the reduction in emissions in pounds of pollutant per day and C is the daily cost in \$ of this reduction.
- a) If a firm is currently reducing its emissions by 10 pounds per day, what is the marginal cost¹ of reducing emission further?
 - b) Government is considering offering a clean air subsidy to the firm based on the formula $S(r) = 500r$ dollars where r is again the daily reduction and S is the subsidy in dollars per day. The government thinks this will result in a reduction in emissions of 50 lbs per day. Are they right?
2. (10 marks) **Foraging bumblebees** Many animals forage on resources that are distributed in discrete patches. For example, bumblebees visit many flowers, foraging nectar from each. The amount of nectar $N(t)$ harvested from a given flower increases the more time is spent at the flower, but the returns diminish, i.e. more nectar is harvested in the first second at the flower when the nectar is most plentiful, than in the second when there is less nectar left. Suppose that these diminishing returns are described by the function,

$$N(t) = \frac{0.3t}{t + 2}$$

Where t is measured in seconds and N in milligrams. Suppose also that it takes a bee 4 seconds on average to travel from one flower to the next.

If a bee spends t seconds per flower then the average amount of nectar it will harvest per second from its arrival at one flower until its arrival at the next is the total nectar it harvests $N(t)$, divided by the time taken, which is the harvest time, t , plus the travel time of 4 seconds.

- a) Write an equation for the average amount of nectar consumed per second, $f(t)$.
- b) What harvesting time per flower would maximize the average rate of energy gain, i.e. maximize the nectar gathered?

¹ Marginal cost is the rate of change in total cost.

3. (10 marks) **Crows and whelks** Coastal crows feed on whelks by flying them up to heights of about 5 m and then dropping them on to rocks to break open their shells, and doing so multiple times if necessary. The author of a study of this phenomenon² asked two questions: 1) “Do crows drop whelks from the best height for breaking?” and 2) “How energetically profitable is dropping of whelks?” To answer these questions whelks were dropped from various heights and a model derived for the number of times n a whelk needs to be dropped from a height h (in meters) to break open

$$n(h) = \frac{h + 14.8}{h - 1.2}$$

Of course there is an energy cost to this activity since the crow has to expend energy to carry the whelk up in the air to drop it. The energy expended is proportional to the height h and the number drops needed $n(h)$.

- What value of h minimizes the energy expended by the crows?
 - The observed average dropping height is 5.3 m. Is this consistent with your answer in a.? What does this indicate about the existence of an optimal foraging strategy?
4. (10 marks) **An economist looks at a tree** The approximate volume in cubic feet of a typical Douglas fir tree can be modelled as a function of its age in years by the equation

$$V = \frac{22,514}{1 + 22,514t^{-2.55}}$$

The lumber can be sold for \$10 per cubic foot, and you do not expect the price to appreciate in the foreseeable future. On the other hand you anticipate an inflation rate of 3% per year, so that the present value of an item that will be worth \$ v in t years' time is given by

$$p = v(1.03)^{-t}$$

- At what age, to the nearest year, should you harvest a Douglas fir tree in order to maximize its present value?
- How much, to the nearest constant dollar, will a Douglas fir tree be worth at that time?
- How does the optimal harvest age vary with the expected rate of inflation?

Hint: Desmos is your friend!

² R. Zach “Decision-making and Optimal Foraging in Northwestern Crows,” Behaviour 28 (1979): 106-117.